MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2014

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with **Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.**
2. The test is of **3** hours duration.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Chemistry, Mathematics** and **Physics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
5. Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

PART-A-CHEMISTRY

1. Consider separate solutions of 0.500 M $C_2H_5OH(aq)$, 0.100 M $Mg_3(PO_4)_2(aq)$, 0.250 M KBr(aq) and 0.125 M $Na_3PO_4(aq)$ at 25°C. Which statement is true about these solutions, assuming all salts to be strong electrolytes?

(A) 0.100 M $Mg_3(PO_4)_2(aq)$ has the highest osmotic pressure.

(B) 0.125 M $Na_3PO_4(aq)$ has the highest osmotic pressure.

(C) 0.500 M $C_2H_5OH(aq)$ has the highest osmotic pressure.

(D*) They all have the same osmotic pressure.

Sol. .5 M C_2H_5OH; i = 1

.1M $Mg_3(PO_4)_2$; i = 5

.25M KBr; i = 2

.125 M Na_3PO_4; i = 4

greater the value of M × i greater will be the osmotic pressure.

∴ (D) All will have same osmotic pressure.

2. Which one of the following bases is not present in DNA?

(A) Adenine (B) Cytosine (C) Thymine (D*) Quinoline

3. Considering the basic strength of amines in aqueous solution, which one has the smallest pk_b value?

(A) CH_3NH_2 (B) $(CH_3)_3N$ (C) $C_6H_5NH_2$ (D*) $(CH_3)_2NH$

Sol. Basic strength of amine is sol.

$(CH_3)_2\ddot{N}H > CH_3\ddot{N}H_2 > (CH_3)_3\ddot{N} > C_6H_5 - \ddot{N}H_2$
(Al L.P. are delocolised)

4. The metal that cannot be obtained by electrolysis of an aqueous solution of its salts is:

(A*) Ca (B) Cu (C) Cr (D) Ag

Sol. The discharge potential of Ca is less than hydrogen, if aqueous solution is taken H_2 will discharged first instead of Ca.

OR

Deposition potential of Cu, Cr and Ag will be more than that of H_2 and deposition potential of Ca will be less than that of H_2.

∴ Ca cannot be obtained by electrolysis.

∴ (1)

5. On heating an aliphatic primary amine with chloroform and ethanolic potassium hydroxide, the organic compound formed is :

(A) an alkanediol (B) an alkyl cyanide (C*) an alkyl isocyanide (D) an alkanol

Sol. Isocyanide test for primary amine

$$R - NH_2 + CHCl_3 \xrightarrow[\Delta]{C_2H_5OH/KOH} \underset{\text{isocyanide (foil smell)}}{R - NC}$$

6. For the equation, $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$

if $K_P = K_C(RT)^x$ where the symbols have usual meaning then the value of x is :

(assuming ideality)

(A*) $\frac{-1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) – 1

Sol. $K_p = K_c\,(RT)^{\Delta n}$

for the given reaction $\Delta n = \frac{1}{2}$

$\therefore$ (1)

7. Given below are the half - cell reactions:

$Mn^{2+} + 2e^- \rightarrow Mn;\ E^o = -1.18\ V$

$2\,(Mn^{3+} + e^- \rightarrow Mn^{2+})\ ;\ E^o = +1.51\ V$

The E^o for $3Mn^{2+} \rightarrow Mn + 2Mn^{3+}$ will be:

(A) – 2.69 V; the reaction will occur

(B) – 0.33 V; the reaction will not occur

(C) – 0.33 V; the reaction will occur

(D*) – 2.69 V; the reaction will not occur

Sol. $Mn^{+2} + 2e^- \rightarrow Mn;$ $E^o = -1.18\ V$

$Mn^{+3} + e^- \rightarrow Mn^{+2};$ $E^o = +1.15\ V$

Given reaction: $3Mn^{+2} \rightarrow Mn + 2Mn^{+3}$ is disproportionation reaction

$\therefore\ E^0 = E^0_{Mn^{+2}/Mn} + E^0_{Mn^{+2}/Mn^{+3}}$

$= -1.18 - 1.51$

$= -2.69\ V$

$\because E^o$ is –ve

$\therefore$ reaction cannot occur at standard condition.

$\therefore$ (D) is the answer.

8. If Z is a compressibility factor, van der Waals equation at low pressure can be written as :

(A*) $Z = 1 - \frac{a}{VRT}$ (B) $Z = 1 - \frac{Pb}{RT}$ (C) $Z = 1 + \frac{Pb}{RT}$ (D) $Z = 1 + \frac{RT}{Pb}$

Sol. $\left(P + \frac{a}{V^2}\right)(v - b) = RT$

at low pressure

V will be very high.

$\therefore$ Volume correction can be neglected.

$\left(P + \frac{a}{V^2}\right)^V = RT$

$\therefore\ PV = RT - \frac{a}{V}$

$\therefore\ Z = 1 - \dfrac{a}{VRT}$

9. In the reaction, $CH_3COOH \xrightarrow{LiAlH_4} A \xrightarrow{PCl_5} B \xrightarrow{AlcKOH} C$ the product C is :

(A) Acetylene (B*) Ethylene (C) Acetyl chloride (D) Acetaldehyde

10. Among the following oxoacids , the correct decreasing order of acid strength is :

(A) $HClO_4 > HOCl > HClO_2 > HClO_3$ (B*) $HClO_4 > HClO_3 > HClO_2 > HOCl$

(C) $HClO_2 > HClO_4 > HClO_3 > HOCl$ (D) $HOCl > HClO_2 > HClO_3 > HOCl_4$

Sol. The stability order of conjugate base is :

$ClO_4^- > ClO_3^- > ClO_2^- > ClO^-$

11. The ratio of masses of oxygen and nitrogen in a particular gaseous mixture is 1: 4. The ratio of number of their molecule is :

(A*) 7: 32 (B) 1: 8 (C) 3: 16 (D) 1: 4

Sol. O_2 and N_2

Let may be x 4x

$\therefore$ moles $\dfrac{x}{32}$ $\dfrac{4x}{28}$

$\therefore$ ratio of molecules will be 7: 32

12. Which one of the following properties is not shown by NO?

(A) It is a neutral oxide

(B) It combines with oxygen to form nitrogen dioxide

(C) It's bond order is 2.5

(D*) It is diamagnetic in gaseous state

Sol. 

Bond order = 2.5

paramagnetic and gas

13. Which series of reactions correctly represents chemical relations related to iron and its compound?

(A) $Fe \xrightarrow{O_2,herat} FeO \xrightarrow{dilH_2SO_4} FeSO_4 \xrightarrow{heat} Fe$

(B) $Fe \xrightarrow{C\ell_2,heat} FeC\ell_3 \xrightarrow{heat,air} FeCl_2 \xrightarrow{Zn} Fe$

(C*) $Fe \xrightarrow{O_2,heat} Fe_3O_4 \xrightarrow{CO,600°C} FeO \xrightarrow{CO,700°C} Fe$

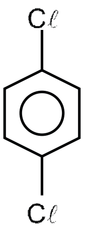
(D) $Fe \xrightarrow{dilH_2SO_4} FeSO_4 \xrightarrow{H_2SO_4,O_2} Fe_2(SO_4)_3 \xrightarrow{heat} Fe$

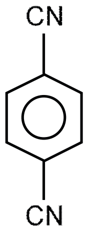
Sol. $Fe + O_2 \xrightarrow{\Delta} Fe$

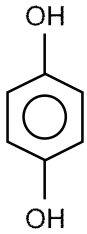
$Fe + O_2 \xrightarrow{\Delta} Fe_2O_3$

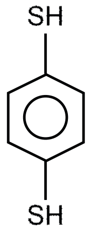
$FeO + Fe_2O_3 \xrightarrow{\Delta} Fe_3O_4$

14. For which of the following molecule significant $\mu \neq 0$?

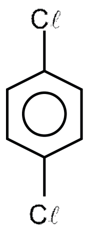
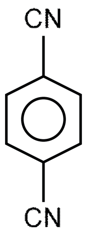
(a) 

(b) 

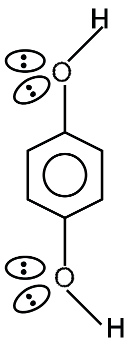
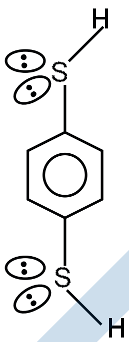
(c) 

(d) 

(A) (a) an (b) (B) only (c) (C*) (c) and (d) (D) only (a)

Sol.  ; 

they are symmetrical and planar so $\mu = 0$

 ; 

so $\mu \neq 0$

15. CsCl crystallises in body centred cubic lattice. If 'a' is its edge length then which of the following expression is correct?

(A) $r_{Cs^+} + r_{Cl^-} = \frac{3a}{2}$ (B*) $r_{Cs^+} + r_{C\ell^-} = \frac{\sqrt{3}\,a}{2}$ (C) $r_{Cs^+} + r_{C\ell^-} = \sqrt{3}\,a$ (D) $r_{Cs^+} + r_{C\ell^-} = 3a$

Sol. $\sqrt{3}a = 2(r_{Cs^+} + r_{C\ell^-})$

16. Which one is classified as a condensation polymer?

(A) Neoprene (B) Teflon (C) Acrylonitrile (D*) Dacron

17. For the estimation of nitrogen, 1.4g of an organic compound was digested by Kjeldahl method and the evolved ammonia was absorbed in 60 mL $\frac{M}{10}$ of sulphuric acid. The unreacted acid required 20 mL of sodium hydroxide for complete neutralization. The percentage of nitrogen in the compound is

(A*) 10% (B) 3% (C) 5% (D) 6%

Sol. $N_{eq}\ NH_3 + n_{eq}\ NaOH = n_{eq}\ H_2SO_4$

or, $a \times 1 + \frac{20 \times \frac{1}{10}}{1000} \times \frac{160 \times \frac{1}{10} \times 2}{1000}$

$\therefore$ a = 0.01

Mass of nitrogen present

$= 0.01 \times 14 = 0.14$gm

$\therefore$ % of N = $\frac{0.14}{1.4} \times 100 = 10\%$

18. For complete combustion of ethanol

$C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$, the amount of heat produced as measured in bomb calorimeter, is 1364.47 kJ mol^{-1} at 25°C. Assuming ideality the Enthalpy of combustion, $\Delta_C H$, for the reaction will be

(R = 8.314 kJ mol^{-1})

(A) – 1361.95 kJ mol^{-1} (B) – 1460.50 kJ mol^{-1} (C) – 1350.50 kJ mol^{-1} (D*) – 1366.95 kJ mol^{-1}

Sol. $\Delta H = \Delta E + \Delta n_g.RT$

$= (-1364.47) + (2 - 3) \times \frac{8.314}{1000} \times 298$

$= -1366.95$ kJ/mol

(No need to solve exactly. Answer may be given directly)

19. The octahedral complex of a metal ion M^{3+} with four monodentate ligands L_1, L_2, L_3 and L_4 absorb wavelengths in the region of red, green, yellow and blue, respectively. The increasing order of ligand strength of the four ligands is:

(A*) $L_1 < L_3 < L_2 < L_4$ (B) $L_3 < L_2 < L_4 < L_1$ (C) $L_1 < L_2 < L_4 < L_3$ (D) $L_4 < L_3 < L_2 < L_1$

Sol. $\xrightarrow{\text{Energy decresing order}}$

V I B G Y O R

↓ ↓ ↓ ↓

L_4 L_2 L_3 L_1

Strength of ligand α energy of light absorbed

$E = \frac{hc}{\lambda}$

$L_1 < L_3 < L_2 < L_4$

20. The most suitable reagent for the conversion of $R{-}CH_2 - OH \rightarrow R - CHO$ is :

(A) $K_2Cr_2O_7$ (B) CrO_3

(C*) PCC (Pyridinium Chlorochromate) (D) $KMnO_4$

Sol. $R - CH_2 - OH \xrightarrow{PCC} RCHO$

other reagent will convert $R - CH_2{-}OH$ into

$R - \underset{\underset{O}{\parallel}}{C} - OH$

21. In which of the following reactions H_2O_2 acts as a reducing agent?

(a) $H_2O_2 + 2H^+ + 2e^- \rightarrow 2H_2O$ (b) $H_2O_2 - 2e^- \rightarrow O_2 + 2H^+$

(c) $H_2O_2 + 2e^- \rightarrow 2OH^-$ (d) $H_2O_2 + 2OH^- - 2e^- \rightarrow O_2 + 2H_2O$

(A) (c), (d) (B) (a), (c) (C*) (b), (d) (D) (a), (b)

Sol. $H_2O_2 - 2e^- \rightarrow O_2 + 2H^+$

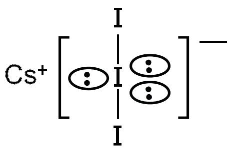
$H_2O_2 + 2OH^- - 2e^- \rightarrow O_2 + 2H_2O$

22. The correct statement for the molecule, CsI_3 is:

(A*) it contains Cs^+ and I_3^- ions. (B) it contains Cs^{3+} and I^- ions.

(C) it contains Cs^+, I^- and lattice I_2 molecule (D) it is a covalent molecule

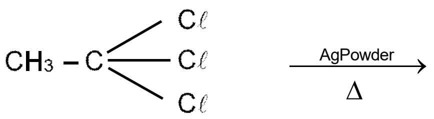
Sol. Cs^+ and I_3^-



23. The major organic compound formed by the reaction of 1,1,1–trichloroethane with silver powder is:

(A) Ethene (B*) 2 – Butyne (C) 2 – Butene (D) Acetylene

Sol.



$CH_3-C\equiv C-CH_3$

24. The equivalent conductance of NaCl at concentration C and at infinite dilution are λ_C and λ_∞, respectively. The correct relationship between λ_C and λ_∞ is given as

(where the constant B is positive)

(A) $\lambda_C = \lambda_\infty - (B)\ C$ (B*) $\lambda_C = \lambda_\infty - (B)\ \sqrt{C}$ (C) $\lambda_C = \lambda_\infty + (B)\ \sqrt{C}$ (D) $\lambda_C = \lambda_\infty + (B)\ C$

Sol. Direct from theory

25. Resistance of 0.2 M solution of an electrolyte is 50Ω. The specific conductance of the solution is 1.4 S m^{-1}. The resistance of 0.5 M solution of the same electrolyte is 280 Ω. The molar conductivity of 0.5 M solution of the electrolyte in S m^2 mol^{-1} is

(A) 5×10^{-3} (B) 5×10^3 (C) 5×10^2 (D*) 5×10^{-4}

Sol. K = 4.4*

For 0.2 M-solution: $1.4 = \frac{1}{50} \times 4^*$ ...(1)

For 0.5 M-solution: $K = \frac{1}{280} \times 4^*$ ...(2)

From (1) / (2), k = Sm^{-1}

Now,

$$\wedge_m = \ = \frac{\frac{1}{4} S\ m^{-1}}{0.5 \times 10^3 mo\ell\ m^{-3}}$$

$= 5 \times 10^{-4}$ S m^2 moℓ^{-1}

OR

Concentration = 0.2

$R_{solution}$ = 50 ohm

K = 1.4 S m^{-1}

Concentration = 0.5

$R_{solution}$ = 280 ohm

$$K = \frac{1}{\text{Resistance}} \text{cell constant}$$

$$\frac{1.4}{100} = \frac{1}{50} \times \text{cell constant}$$

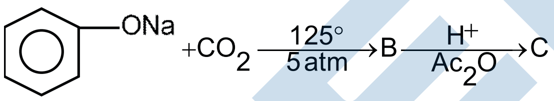
0.7 = cell constant

$$\wedge_m = \frac{\left(\frac{1}{280} \times 0.7\right) \times 1000}{0.5}$$

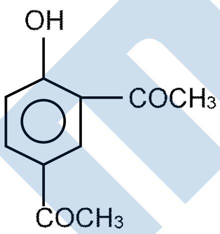
$$= \frac{700 \times 2}{280} = \text{S cm}^2/\text{m}\Omega$$

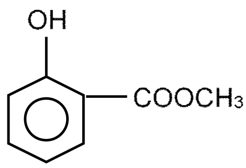
$= 5 \times 10^{-4}$ S m^2/mΩ

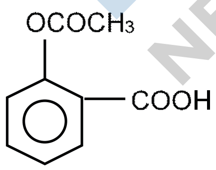
26. Sodium phenoxide when heated with CO_2 under pressure at 125°C yields a product which on acetylation produces C.

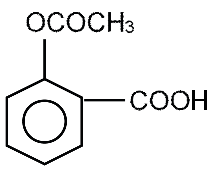


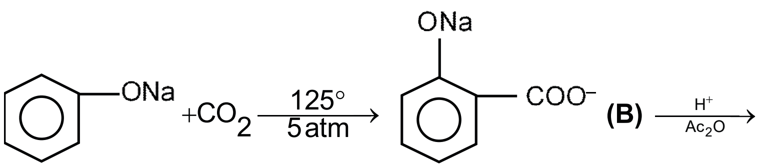
The major product C would be:

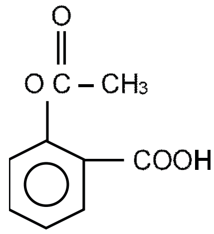
(A) 

(B) 

(C) 

(D*) 

Sol. 



(C)

Aspirin

(Analgesic and blood diluter)

Most wildly used drub would wide

27. The equation which is balanced and represents the correct product(s) is :

(A*) $[CoCl(NH_3)_5]^+ + 5H^+ \rightarrow Co^{2+} + 5NH_4^+ + Cl^-$

(B) $[Mg(H_2O_6)]^{2+} + (EDTA)^{4-} \xrightarrow{\text{excessNaOH}} [Mg(EDTA)]^{2+} + 6H_2O$

(C) $CuSO_4 + 4KCN \rightarrow K_2[Cu(CN)_4] + K_2SO_4$

(D) $Li_2O + 2KCl \rightarrow 2LiCl + K_2O$

Sol. $[CoCl(NH_3)_5]^+ + 5H^+ \rightarrow Co^{2+} + 5NH_4^+ + Cl^-$

$[CoCl(NH_3)_5]^+$ is imperfect complex with respect to acid

28. The correct set four quantum numbers for the valence electrons of rubidium atom (Z = 37) is :

(A) 5, 1, 0, $+\frac{1}{2}$ (B) 5, 1, 1, $+\frac{1}{2}$ (C) 5, 0, 1, $+\frac{1}{2}$ (D*) 5, 0, 0, $+\frac{1}{2}$

Sol. Rubidium atom (Z = 37) = $[Kr]\ 5s^1$

36

$n = 5,\ \ell = 0,\ m = 0,\ s = +\frac{1}{2}$

29. For the non-stoichiometre reaction $2A + B \rightarrow C + D$, the following kinetic data were obtained in three separate experiments, all at 298 K.

Initial Concentration (A)	Initial Concentration (B)	Initial rate of formation of C $(mol\ L^{-}S^{-})$
0.1 M	0.1 M	1.2×10^{-3}
0.1 M	0.2 M	1.2×10^{-3}
0.2 M	0.1 M	2.4×10^{-3}

The rate law for the formation of C is

(A) $\frac{dc}{dt} = k\,[A]^2[B]$ (B) $\frac{dc}{dt} = k\,[A]\,[B]^2$ (C*) $\frac{dc}{dt} = k[A]$ (D) $\frac{dc}{dt} = k[A]\,[B]$

Sol. Rate = $k[A]^x\,[B]^y$

$$\left[\frac{A_1}{A_2}\right]^x \left[\frac{B_1}{B_2}\right]^y$$

From 1 and 2

$$\frac{1.2\times10^{-3}}{1.2\times10^{-3}}=\left[\frac{0.1}{0.1}\right]^x\left[\frac{0.1}{0.2}\right]^y$$

y = 0

From 1 and 3

$$=\frac{1.2\times10^{-3}}{2.4\times10^{-3}}=\left[\frac{0.1}{0.2}\right]^x\left[\frac{0.1}{0.1}\right]^y$$

x = 1

Rate = $\frac{dc}{dt}$ = k[A]

30. In S_N2 reactions, the correct order of reactivity for the following compounds:

CH_3Cl, CH_3CH_2Cl, $(CH_3)_2$ CHCl and $(CH_3)_3CCl$ is:

(A*) $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3$ CCl

(B) $CH_3CH_2Cl > CH_3Cl > (CH_3)_2$ CHCl > $(CH_3)_3CCl$

(C) $(CH_3)_2CHCl > CH_3CH_2Cl > CH_3Cl > (CH_3)_3CCl$

(D) $CH_3Cl > (CH_3)_2$ CHCl > $CH_3CH_2Cl > (CH_3)_3CCl$

Sol. Order rate durig S_N2 reaction

$$\alpha\frac{1}{\text{steric hindrence}}$$

so $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3$ CCl

PART-B-MATHEMATICS

31 $\lim\limits_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is equal to

(A*) π (B) $\frac{\pi}{2}$ (C) 1 (D) $-\pi$

Sol. $\lim\limits_{x \to 0} \frac{\sin\left(\pi(1-\sin^2 x)\right)}{x^2}$

$= \lim\limits_{x \to 0} \left(\frac{\sin(\pi \sin^2 x)}{\pi \sin^2 x}\right) \frac{\pi \sin^2 x}{x^2} = \pi$ **Ans.**

32. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}$ = p(t) – 200.

If p(0) = 100, then p(t) equals

(A) 400 – 300 (B*) 400 – 300 (C) 300 – 200 (D) 600 – 500

Sol. $\frac{dp(t)}{dt} + \left(\frac{-1}{2}\right) pt = -200$

I.F. = $e^{-\int \frac{1}{2} dt} = e^{\frac{-t}{2}}$

$P(t)\, e^{-t/2} = \int (-200) e^{\frac{-t}{2}}\, dt + C$

$= \frac{-200\, e^{-\frac{t}{2}}}{-\frac{1}{2}} + C$

$P(t)\, e^{-t/2} = 400\, e^{-t/2} + C$

100 = 400 + C

C = – 300

$P(t) = 400 - 300\, e^{t/2}$

33. The statement $\sim (p \leftrightarrow \sim q)$ is

(A) a fallacy (B*) equivalent to $p \leftrightarrow q$

(C) equivalent to $\sim p \leftrightarrow q$ (D) a tautology

Sol.

p	q	~ q	(p ↔~ q)	~ (p ↔~ q)	p ↔ q
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

34. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is

(A*) $\frac{2\sqrt{13}}{9}$ (B) $\frac{\sqrt{61}}{9}$ (C) $\frac{2\sqrt{17}}{9}$ (D) $\frac{\sqrt{34}}{9}$

Sol. $px^2 + qx + r = 0 <^{\alpha}_{\beta}$, $p \neq 0$

Given, p, q, r are in A.P.

So, $2q = p + r$ .... (i)

and $\frac{\alpha+\beta}{\alpha\beta} = 4 \Rightarrow \frac{-q}{p} = \frac{4r}{p} \Rightarrow q+4r = 0$

Also, $-8r = p + r \Rightarrow p = -9r$ {from (i)}

Now, $|\alpha - \beta|^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= \frac{q^2}{p^2} - \frac{4r}{p} = \frac{q^2 - 4pr}{p^2} = \frac{16r^2 - 4pr}{p^2}$

$= \frac{16 \times p^2}{81p^2} - \frac{4}{p}\left(\frac{-p}{9}\right) = \frac{16}{81} + \frac{4}{9}$

$= \frac{16+36}{81} = \frac{52}{81}$

Hence, $|\alpha - \beta| = \sqrt{\frac{52}{81}} = \frac{2\sqrt{13}}{9}$. **Ans.**

35. Let PS be the median of the triangle with vertices P(2, 2), Q(6, – 1) and R (7, 3). The equation of the line passing through (1, – 1) and parallel to PS is

(A) $2x - 9y - 11 = 0$ (B) $4x - 7y - 11 = 0$ (C*) $2x + 9y + 7 = 0$ (D) $4x + 7y + 3 = 0$

Sol. $S\left(\frac{13}{2}, 1\right)$

$m_{PS} = \frac{2-1}{2-\frac{13}{2}} = \frac{1}{-9/2} = \frac{-2}{9}$

So, equation of line is

$$(y + 1) = \frac{-2}{9}(x - 1)$$

$\Rightarrow 9y + 9 = -2x + 2$

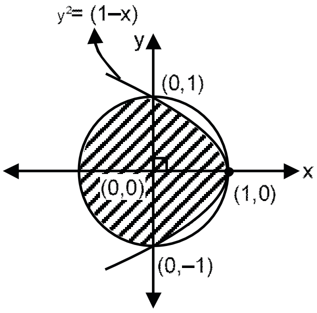
$\Rightarrow 2x + 9y + 7 = 0$. **Ans.**

36. The area of the region described by

$A = \{(x, y): x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is

(A) $\frac{\pi}{2}+\frac{2}{3}$ (B*) $\frac{\pi}{2}+\frac{4}{3}$ (C) $\frac{\pi}{2}+\frac{4}{3}$ (D) $\frac{\pi}{2}-\frac{2}{3}$

Sol.



Shaded area $= \frac{\pi}{2} + 2\left(\int_0^1 \sqrt{1-x}\,dx\right)$

$= \frac{\pi}{2} + 2\left(\frac{2}{3}\right) = \frac{\pi}{2} + \frac{4}{3}$

37. If A is an 3 × 3 non-singular matrix such that AA' = A'A and B = $A^{-1}A'$, then BB' equals

(A) $(B^{-1})'$ (B) I + B (C*) I (D) B^{-1}

Sol. $BB^T = A^{-1}A^T (A^{-1}A^T)^T$

$= A^{-1}A^T A(A^{-1})^T = A^{-1}AA^T(A^{-1})^T$

$= A^T (A^{-1})^T = (A^{-1}A)^T = I^T = I$. **Ans.**

38. The image of the line

$\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane

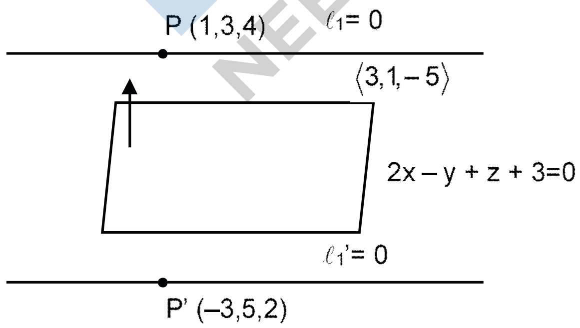
2x – y + z + 3 = 0 is the line

(A) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$ (B*) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

(C) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$ (D) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

Sol. Line is parallel to the given plane

$\ell_1' \frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$



39. If x = – 1 and x = 2 are extreme points of f(x) = α log | x | + βx^2 + x, then

(A) $\alpha = 2, \beta = \frac{1}{2}$ (B) $\alpha = -6, \beta = \frac{1}{2}$ (C) $\alpha = -6, \beta = \frac{-1}{2}$ (D*) $\alpha = 2, \beta = \frac{-1}{2}$

Sol. $f'(x) = \frac{\alpha}{x} + 2\beta x + 1$

Now, $f'(-1) = 0$

$\Rightarrow -\alpha - 2\beta + 1 = 0$ ...... (1)

and $f'(2) = 0$

$\Rightarrow \frac{\alpha}{2} + 4\beta + 1 = 0$ ...... (2)

$\therefore$ On solving, we get

$\alpha + 2\beta = 1$

$\alpha + 8\beta = -2$

$6\beta = -3 \Rightarrow \beta = \frac{-1}{2}$,

Now, $\alpha = 1 - 2\beta = 2$

Hence, $\alpha = 2,\ \beta = \frac{-1}{2}$ **Ans.**

40. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$(where [x] denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval

(A) $(-\infty, -2) \cup (2, \infty)$ (B*) $(-1, 0) \cup (0, 1)$ (C) $(1, 2)$ (D) $(-2, -1)$

Sol. Let $x - [x] = f$

$\therefore 3f^2 - 2f - a^2 = 0$ ....(1)

$f = \frac{1 + \sqrt{1 + 3a^2}}{3}$

for no integral solution

$\Rightarrow 0 < f < 1 \Rightarrow 0 < \frac{1 + \sqrt{1 + 3a^2}}{3} < 1$

$\Rightarrow 0 \leq a^2 < 1$

$\therefore a \in (-1, 1)$

But if a = 0, then from equation (i), f = 0

But it will give integral value of x.

$\therefore$ a = 0 is not possible.

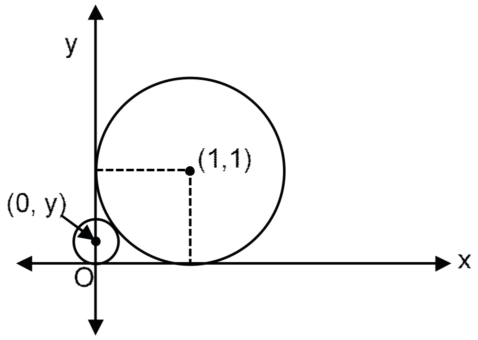
So, $a \in (-1, 1) - \{0\}$. **Ans.**

41. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y) passing through origin and touching the circle C externally, then the radius of T is equal to

(A*) $\frac{1}{4}$ (B) $\frac{\sqrt{3}}{\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

Sol. Let required circle is

$(X - 0)^2 + (Y - y)^2 = y^2$



$\because$ circle touches externally

$\therefore C_1C_2 = r_1 + r_2$

$\therefore (1 - 0)^2 + (1 - y)^2 = (y + 1)^2$

$\therefore\ y = \frac{1}{4}$. **Ans.**

42. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to

(A*) $\left(16, \frac{272}{3}\right)$ (B) $\left(16, \frac{251}{3}\right)$ (C) $\left(14, \frac{251}{3}\right)$ (D) $\left(14, \frac{272}{3}\right)$

Sol.$_{bin}$ coefficient of x^3 in $(1 + ax + bx^2)(1 - 2x)^{18}$

= coefficient of x^3 in $(1 - 2x)^{18}$

+ a · coefficient of x^2 in $(1 - 2x)^{18}$

+ b · coefficient of x in $(1 - 2x)^{18} = 0$

$\Rightarrow {}^{18}C_3(-2)^3 + a \cdot {}^{18}C_2(-2)^2$

$+ b \cdot {}^{18}C_1(-2)^1 = 0$ .... (1)

and

coefficient of x^4 in $(1 + ax + bx^2)(1 - 2x)^{18}$

= coefficient of x^4 in $(1 - 2x)^{18}$

+ a · coefficient of x^3 in $(1 - 2x)^{18}$

+ b · coefficient of x^2 in $(1 - 2x)^{18} = 0$

$\Rightarrow {}^{18}C_4(-2)^4 + a \cdot {}^{18}C_3(-2)^3$

$+ b \cdot {}^{18}C_2(-2)^2 = 0$ ....(2)

$\therefore$ From (1) and (2)

we get, a = 16, b = $b = \frac{272}{3}$. **Ans.**

43. If z is complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$

(A) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$ (B) is equal to $\frac{5}{2}$

(C*) lies in the interval (1, 2) (D) is strictly greater than $\frac{5}{2}$

Sol. $\left|z-\left(\frac{-1}{2}\right)\right|$

= distance of z from point $\left(\frac{-1}{2}\right)$

= $\frac{3}{2}$ is its minimum value and it goes to infinity.

44. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}\,dx$ is equal to

(A) $-xe^{\left(x+\frac{1}{x}\right)}+C$ (B) $(x-1)e^{\left(x+\frac{1}{x}\right)}+C$

(C*) $x\,e^{\left(x+\frac{1}{x}\right)}+C$ (D) $(x+1)e^{\left(x+\frac{1}{x}\right)}+C$

Sol. We know that $\int(\,f(x)+xf'(x)\,)\,dx = x\,f(x)$

So, $\int\left(e^{\left(x+\frac{1}{x}\right)}+\left(x-\frac{1}{x}\right)e^{\left(x+\frac{1}{x}\right)}\right)dx$

Now, $f(x) = e^{\left(x+\frac{1}{x}\right)}$

$\therefore$ $x\,f(x) = x\,e^{\left(x+\frac{1}{x}\right)}$. **Ans.**

45. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

(A) $\frac{2}{3}$ (B*) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $\frac{1}{8}$

Sol. $y = mx + \frac{1}{m}$ is tangent to parabola $x^2 = -32y$ also

$\therefore x^2 = -32\left(mx+\frac{1}{m}\right)$

$x^2 + 32mx + \frac{32}{mk} = 0$

its D = 0 gives m = $\frac{1}{2}$

46. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals

(A*) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Sol. $f_4(x) - f_6(x)$

$= \frac{1}{4}\left(1-\frac{\sin^2 2x}{2}\right)-\frac{1}{6}\left(1-\frac{3}{4}\sin^2 2x\right)$

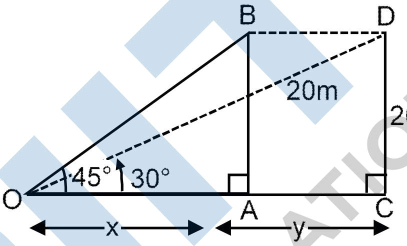
$= \frac{1}{4}\left(1-\frac{\sin^2 2x}{2}\right)-\frac{1}{6}\left(1-\frac{3}{4}\sin^2 2x\right) = \frac{1}{12}$

47. A bird is sitting on the top of a vertical pole 20 m high and its elevation from the point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is

(A*) $20\left(\sqrt{3}-1\right)$ (B) $40\left(\sqrt{2}-1\right)$ (C) $40\left(\sqrt{3}-\sqrt{2}\right)$ (D) $20\sqrt{2}$

Sol. In ΔAOB

$\tan 45° = \frac{20}{x} \Rightarrow x = 20$



In ΔDOC

$\tan 30° = \frac{20}{x+y}$

$\frac{1}{\sqrt{3}} = \frac{20}{20+V}$

where V is speed in m/s

$V = 20(\sqrt{3}-1)\ m/s$

48. If $\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right] = \lambda[\vec{a}\vec{b}\vec{c}]^2$ then λ is equal to

(A*) 1 (B) 2 (C) 3 (D) 0

Sol. As, $\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right] = [\vec{a}\vec{b}\vec{c}]^2$.

So, $\lambda = 1$

49. Let A and B be two events such that $P\left(\overline{A\cup B}\right)=\frac{1}{6}$, $P(A\cap B)=\frac{1}{4}$ and $P(\bar{A})=\frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are

(A) independent and equally likely (B) mutually exclusive and independent

(C) equally likely but not independent (D*) independent but not equally likely

Sol. $P(A\cup B) = \frac{5}{6}; P(A) = \frac{3}{4}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$

$P(B) = \frac{1}{3}$

Now, $P(A\cap B) = P(A).P(B)$

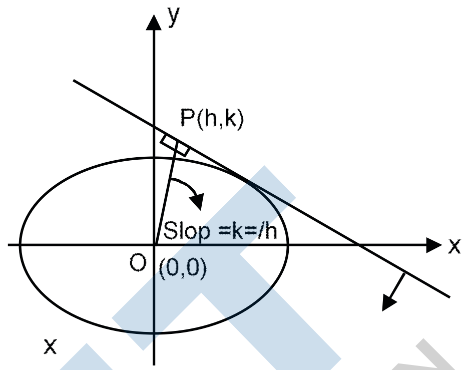
Hence, A and B are independent events

50. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is

(A) $(x^2 + y^2)^2 = 6x^2 - 2y^2$ (B) $(x^2 - y^2)^2 = 6x^2 + 2y^2$

(C) $(x^2 - y^2)^2 = 6x^2 - 2y^2$ (D*) $(x^2 + y^2)^2 = 6x^2 + 2y^2$

Sol.



T: y = mx ±

$(k - mh)^2 = 6m^2 + 2$

where $m = \frac{-h}{k}$

$\left(k + \frac{h^2}{k}\right)^2 = \frac{6h^2}{k^2} + 2$

$(x^2 + y^2)^2 = 6x^2 + 2y^2$

51. If f and g are differentiable functions in [0, 1] satisfying f (0) = 2 = g (1), g(0) = 0 and f (1) = 6, then for some $c \in]0, 1[$

(A*) f ' (c) = 2 g ' (c) (B) 2 f ' (c) = g ' (c) (C) 2 f ' (c) = 3 g ' (c) (D) f ' (c) = g ' (c)

Sol. Let h(x) = f (x) – 2g(x)

Apply Rolle's theorem to h(x) in [0, 1].

52. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is

(A*) $2+\sqrt{3}$ (B) $\sqrt{2}+\sqrt{3}$ (C) $3+\sqrt{2}$ (D) $2-\sqrt{3}$

Sol. Let 3 positive numbers in GP be

a, ar, ar^2 where r > 1

Now, a, 2ar, ar^2 are in AP

$4ar = a + ar^2$

$r^2 - 4r + 1 = 0$

$r = \frac{4 \pm \sqrt{16-4}}{2}$

$r = 2 \pm \sqrt{3}$ but r > 1

$\therefore r = 2 + \sqrt{3}$

53. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then g ' (x) is equal to

(A*) $1 + \{g(x)\}^5$ (B) $1 + x^5$ (C) $5x^4$ (D) $\frac{1}{1+\{g(x)\}^5}$

Sol. $g'(y) = \frac{1}{f'(x)} = 1 + x^5$

$g'(x) = 1 + (g(x))^5$

54. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines 4ax + 2ay + c = 0 and 5bx + 2by + d = 0 lies in the fourth quadrant and is equidistant from the two axes then

(A) 3bc + 2ad = 0 (B) 2bc – 3ad = 0 (C) 2bc + 3ad = 0 (D*) 3bc – 2ad = 0

Sol. Solving both the lines get

$\frac{bc-ad}{ab}$, $y = \frac{4ad-5bc}{2ab}$

(Using cross multiplication method)

Now, x + y = 0 $\Rightarrow$ 2ad – 3bc = 0

55. If $\alpha, \beta \neq 0$, and f (n) = $\alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2,$$

then K is equal to

(A) – 1 (B) $\alpha\beta$ (C) $\frac{1}{\alpha\beta}$ (D*) 1

Sol. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$

(Using product of two determinants)

= $(1-\alpha)^2(\alpha-\beta)^2(\beta-1)^2$

$\therefore$ k = 1

56. The integral equals $\int_0^{\pi} \sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$

(A*) $4\sqrt{3}-4-\frac{\pi}{3}$ (B) $\pi - 4$ (C) $\frac{2\pi}{3}-4-4\sqrt{3}$ (D) $4\sqrt{3}-4$

Sol. $\int_0^{\pi} \left|1-2\sin\frac{x}{2}\right| dx$

Put, $\frac{x}{2} = t \Rightarrow dx = 2dt$

$2\int_0^{\frac{\pi}{2}} |1-2\sin t|\,dt$

$$= 2\left(\int_0^{\frac{\pi}{6}}(1-2\sin t)dt + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(2\sin t-1)dt\right) = 4\sqrt{3}-4-\frac{\pi}{3}$$

57. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots\ldots\ldots\ldots + 10(11)^9 = k(10)^9$, then k is equal to

(A) 110 (B) $\frac{121}{10}$ (C) $\frac{441}{100}$ (D*) 100

Sol. $10^9\left(1+2.\left(\frac{11}{10}\right)+3.\left(\frac{11}{10}\right)^2+....+10.\left(\frac{11}{10}\right)^9\right)$

Let k = 1 + 2x + 3x^2 + ... + 10x^9

where $x=\frac{11}{10}$

$k=\frac{1-x^{10}}{(1-x)^2}-10.\frac{x^{10}}{1-x}$

k = 100

58. The angle between the lines whose direction cosines satisfy the equations ℓ + m + n = 0 and ℓ^2 = m^2 + n^2 is

(A) $\frac{\pi}{2}$ (B*) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Sol.$_{3\text{-d}}$ ℓ = –m – n

$(m + n)^2 = m^2 + n^2 \Rightarrow 2mn = 0$

m = 0, ℓ = –n

n = 0, ℓ = m

Dr's are : –n, 0, n and –m, m, 0

or : –1, 0, 1 and –1, 1, 0

$\cos\theta=\frac{1}{\sqrt{2}.\sqrt{2}}\Rightarrow\theta=\frac{\pi}{3}$

59. The variance of first 50 even natural numbers is

(A) $\frac{437}{4}$ (B) $\frac{833}{4}$ (C*) 833 (D) 437

Sol. $\sigma^2=\frac{n^2-1}{3}$

$=\frac{(50)^2-1}{3}=833$

60. If X = {4^n – 3n – 1 : n $\in$ N}} and Y = {9(n – 1) : n $\in$ N}, where N is the set of natural numbers, then X $\cup$ Y is equal to

(A*) Y (B) N (C) Y – X (D) X

Sol. X = {0, 9, 54, 243 ……, ……}

Y = {0, 9, 18, 27……, ……}

X $\cup$ Y = Y.

[Note that : $4^n - 3n - 1 = (1 + 3)^n - 3n - 1$

$= (1 + 3n + {}^nC_2 \cdot 3^2 + {}^nC_3 \cdot 3^3 + ...)$

$- 3n - 1 = {}^nC_2 \cdot 3^2 + {}^nC_3 \cdot 3^3 + ...,$

which is multiple of 9.

Here, X $\cap$ Y = X.

PART-C-PHYSICS

61. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is:

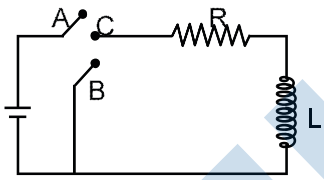
(For steel Young's modulus is 2×10^{11} Nm^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})

(A) 2.2×10^9 Pa (B) 2.2×10^7 Pa (C) 2.2×10^6 Pa (D*) 2.2×10^8 Pa

Sol. $= y\alpha\Delta T$

$= 2 \times 10^8 \times 1.1 \times 10^{-5} \times 100$

62. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time t = 0. Ratio of the voltage across resistance and the inductor at t = L/R will be equal to :

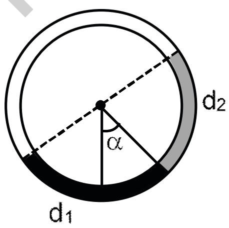


(A) 1 (B*) –1 (C) $\frac{1-e}{e}$ (D) $\frac{e}{1-e}$

63. The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to :

(A*) 1.1 eV (B) 0.8 eV (C) 1.6 eV (D) 1.8 eV

64. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is:

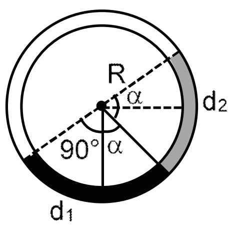


(A) $\frac{1+\cos\alpha}{1-\cos\alpha}$ (B*) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (C) $\frac{1+\sin\alpha}{1-\cos\alpha}$ (D) $\frac{1+\sin\alpha}{1-\sin\alpha}$

Sol. $d_2 \times A \sin\alpha + \frac{d}{2} \times R\cos\alpha$

$= d_1 (R\cos\alpha - R\cos\alpha)$

$\frac{d_2}{d_1} = \frac{\cos\alpha - \sin\alpha}{\sin\alpha + \cos\alpha}$



65. A bob of mass m attached to an inextensible string of length ℓ is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension:

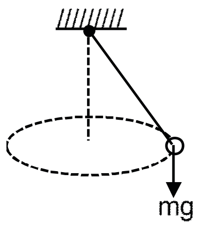
(A) angular momentum changes in magnitude but not in direction.

(B*) angular momentum changes in direction but not in magnitude.

(C) angular momentum changes both in direction and magnitude.

(D) angular momentum is conserved.

Sol.



τ of my changes direction but not magnitude.

66. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is :

(A*) $f_1 > f$ and f_2 becomes negative

(B) $f_2 > f$ and f_1 becomes negative

(C) f_1 and f_2 both become negative

(D) $f_1 = f_2 < f$

Sol. $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{8}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{-1}{10}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

67. A green light is incident from the water to the air-water interface at the critical angle (θ). Select the correct statement.

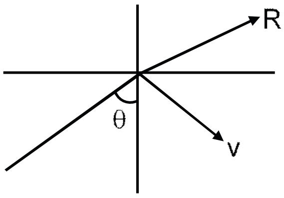
(A*) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

(B) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.

(C) The entire spectrum of visible light will come out of the water at various angles to the normal.

(D) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal.

Note: **The question is wrong because green light will not give any spectrum. Answer is given assuming incident light to be white.**

Sol. 

$\mu \sin \theta = 1$

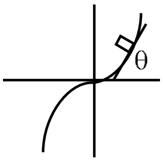
(Assuming incident light is white)

$\mu_V > \mu_R$

$q_V < q_R$

68. A block of mass m is placed on a surface with a vertical cross-section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is:

(A) $\frac{2}{3}$m (B) $\frac{1}{3}$m (C) $\frac{1}{2}$m (D*) $\frac{1}{6}$m

Sol. 

$\mu = \tan\theta = 0.5 = \frac{dy}{dx} = \frac{x^2}{2}$

$x^2 = 1$

$x = 1 \quad \Rightarrow y = \frac{1}{6}$m

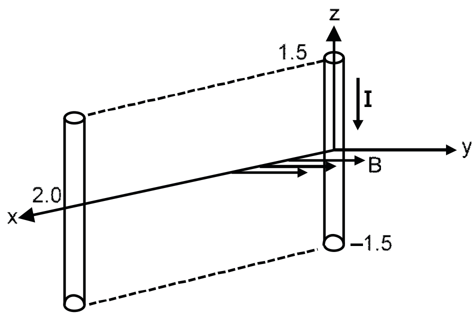
69. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is:

(A) 60 mA (B*) 3 A (C) 6 A (D) 30 mA

Sol. $3 \times 10^3 = x_i = \frac{100}{0.1} \times i$

i = 3A

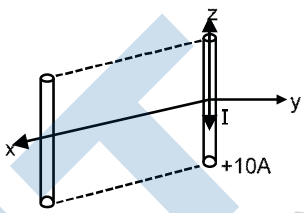
70. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0 \times 10^{-4}\ e^{-0.2x}\ \hat{a}_y$ T, find the power required to move the conductor at constant speed to x = 2.0 m, y = 0 m in 5×10^{-3} s. Assume parallel motion along the x-axis.



(A*) 2.97 W (B) 14.85 W (C) 29.7 W (D) 1.57 W

Sol. $F = i\ell B$

$= 10 \times 3 \times 3 \times 10^{-4} \times e^{-0.2x}$



$$\int dW = 9 \times 10^{-3} \int_0^2 e^{0.2x} dx$$

$$P_{av} = \frac{9 \times 10^{-3} \times \left.\dfrac{e^{0.2x}}{-0.2}\right|_0^2}{t}$$

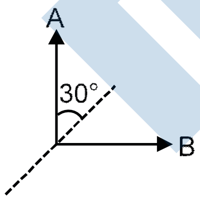
$$= \frac{9 \times 10 \times 10^{-3}}{5 \times 10^{-3} \times 2}\left[1 - e^{-0.4}\right]$$

$$= 9\left[1 - \frac{1}{e^{2/5}}\right] = 2.97 \text{ W}$$

71. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\frac{I_A}{I_B}$ equals:

(A) $\frac{3}{2}$ (B) 1 (C*) $\frac{1}{3}$ (D) 3

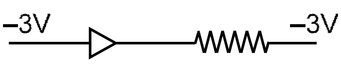
Sol.



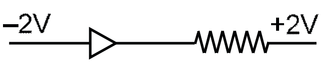
$I_A \cos^2 30° = I_B \cos_2 60°$

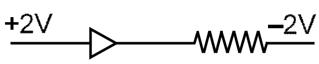
$$\frac{I_A}{I_B} = \frac{1}{3}$$

72. The forward biased diode connection is:

(A)


(B)


(C)


(D*)


73. Assume that an electric field exists in space. Then the potential difference $V_A - V_O$, where V_0 is the potential at the origin and V_A the potential at x = 2 m is

(A) –120 J (B*) –80 J (C) 80 J (D) 120 J

Note: The question is wrong because the unit of potential difference is volt.

Sol. $E = -\frac{dU}{dx} = 30x^2$

$$DU = \int dU = -30\int_0^2 x^2 dx = -10[8 - 0]$$

= –80 J

74. Match List-I (Electromagnetic wave type) with List-II (Its association/application) and select the correct option from the choices given below the lists:

List-I	List-II
(a) Infrared waves	(i) To treat muscular strain
(b) Radio waves	(ii) For broadcasting
(c) X-rays	(iii) To detect fracture of bones
(d) Ultraviolet rays	(iv) Absorbed by the ozone layer of the atmosphere

	a	b	c	d
(A)	(i)	(ii)	(iv)	(iii)
(B)	(iii)	(ii)	(i)	(iv)
(C*)	(i)	(ii)	(iii)	(iv)
(D)	(iv)	(iii)	(ii)	(i)

Sol. $\frac{dI}{dV} = e^{\frac{10V}{3}} \times \frac{10}{3}$

$S = e^{\frac{10V}{3}} - 1$

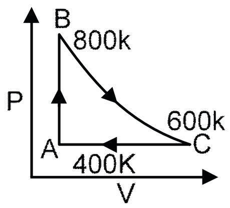
$e^{\frac{10V}{3}} = 6$

dI = 0.2 mA

75. The current voltage relation of diode is given by I = $(e^{1000v/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree kelvin. If a student makes an error measuring ±0.01 V while measuring the current of 5mA at 300K, what will be the error in the value of current in mA?

(A) 0.02 mA (B) 0.5 mA (C) 0.05 mA (D*) 0.2 mA

76. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperature at A, B and C are 400K, 800K and 600K respectively. Choose the correct statement :



(A) The change in internal energy in the process CA is 700 R

(B) The change in internal energy in the process AB is –350 R

(C*) The change in internal energy in the process BC is –500 R

(D) The change in internal energy in whole cyclic process is 250 R

Sol. $\Delta U_{CA} = \frac{5}{3}R\ [200]$

$\Delta U_{AB} = \frac{5}{2}R\ [400]$

$\Delta U_{BC} = \frac{5}{2}R \times -100 = -500$

77. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s.

(A) 8 (B*) 6 (C) 4 (D) 12

Sol. $f = \frac{2x+1}{4L \times 0.85} \times 340$

= 100, 300, 500, 700, 900, 1100, ...

78. In a large building, there are 15 bulbs of 40W, 5 bulbs of 100W, 5 fans of 80W and 1 heater of 1kW. The voltage of the electric mains is 220V. The minimum capacity of the main fuse of the building will be

(A) 10 A (B*) 12 A (C) 14 A (D) 8A

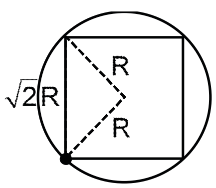
Sol. P = Vi

$i = \frac{40 \times 15}{220} + \frac{100 \times 5}{220} + \frac{80 \times 5}{220} + \frac{1000}{220}$

$= \frac{125}{11} = 11A$

79. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is :

(A) $\sqrt{2\sqrt{2}\frac{GM}{R}}$ (B) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (C*) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (D) $\frac{1}{2}\sqrt{\frac{GM}{R}}(1+2\sqrt{2})$



Sol.

$\frac{mv^2}{R} = \frac{Gm^2}{(\sqrt{2}R)^2}\cos 45° \times 2 + \frac{Gm^2}{(2R)^2}$

$= \frac{Gm}{\sqrt{2}R} + \frac{Gm}{4R}$

$\frac{Gm}{4R}[1 + 2\sqrt{2}]$

80. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is

(A) $g H = (n - 2)^2 u^2$

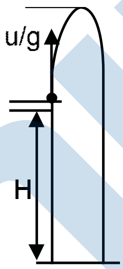
(B*) $2g H = nu^2(n - 2)$

(C) $g H = (n - 2)u^2$

(D) $2 g H = n^2 u^2$

Sol. $-H = ut - \frac{-1}{2}gt^2$

$\frac{1}{2} gt^2 - ut - H = 0$



$\frac{nu}{g} = t = \frac{ut\sqrt{u^2 + 2gM}}{g}$

$(n - 1)u = \sqrt{u^2 + 2gH}$

$(n - 1)^2 u^2 - u^2 = 2gH$

$u^2[n^2 - 2n] = 2gH$

81. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

(A*) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.

(B) A screw gauge having 100 divisions in the circular scale and pitch as 1mm.

(C) A screw gauge having 50 divisions in the circular scale and pitch as 1mm

(D) A meter scale.

82. A parallel plate capacitor is made of two circular plates separated by a distance of 5mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to

(A) 3×10^{-7} C/m^2 (B) 3×10^4 C/m^2 (C) 6×10^4 C/m^2 (D*) 6×10^{-7} C/m^2

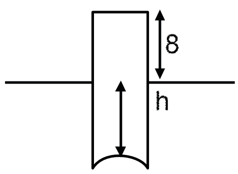
Sol. $E = \frac{\sigma}{h \in_0}$

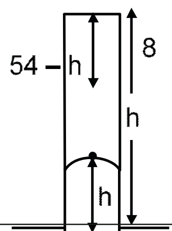
$\sigma = 3 \times 10^4 \times 2.2 \times 8.85 \times 10^{-12}$

$= 6 \times 10^{-7}$

$76 \times (8 + h_0) = (76 - h - h_0)(54 - h)$

$76 \times 8 + 76h_0 = 76 \times 54 + h^2$





or

$E = \frac{\sigma}{h \in_0}$

$\Rightarrow 3 \times 10^4 = \frac{\sigma}{2.2 \times 8.85 \times 10^{-12}}$

83. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be the length of the air column above mercury in the tube now ? (Atmospheric pressure = 76 cm of Hg)

(A) 22 cm (B) 38 cm (C) 6 cm (D*) 16 cm

Sol. 76 × 8 = (76 + x – 54)x

84. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then

(A) time period of oscillations is 8τ

(B) amplitude of motion is 4a

(C*) time period of oscillations is 6τ

(D) amplitude of motion is 3a

Sol. R – cos θ = a

R – R cos 2θ = 3a

$\frac{1-\cos\theta}{1-\cos 2\theta} = \frac{1}{3}$

$\frac{1-\cos\theta}{2\sin^2\theta} = \frac{1}{3}$

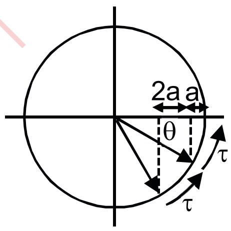
3 – 3 cosθ = 2 – 2 cos²θ

2 cos²θ – 3 cosθ + 1 = 0 ∴ θ = 60º

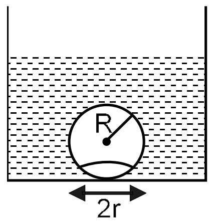
2cos²θ – 2cosθ – cosθ + 1 = 0

2cosθ (cos θ – 1) – 1

θ = 60º



85. On heating water, bubbles being formed at the bottom of the vessel detatch and rise. Take the bubbles to be sphere of radius R and making a circular contact of radius r with the bottom of the vessel. If r << R, and the surface tension of water is T, value of r just before bubbles detatch is : (density of water is ρ_w)



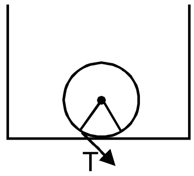
(A) $R^2\sqrt{\frac{\rho_w g}{6T}}$ (B) $R^2\sqrt{\frac{\rho_w g}{T}}$ (C) $R^2\sqrt{\frac{3\rho_w g}{T}}$ (D*) $R^2\sqrt{\frac{\rho_w g}{3T}}$

Ans. [None of these] Ans. is $R^2\sqrt{\frac{2\rho_w g}{3T}}$

Sol. $\frac{4}{3}\pi R^3 - \rho_w g$

$= T \times 2\pi r \sin\theta$

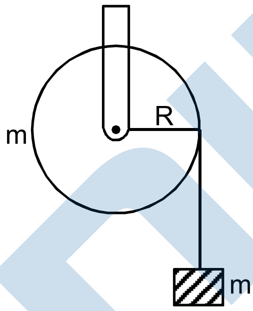
$= T \times \frac{2\pi r^2}{R}$



$r = \sqrt{\frac{2}{3}\frac{R^4 \rho_w g}{T}}$

$= R^2 = \sqrt{\frac{2}{3}\frac{\rho_w g}{T}}$

86. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release ?



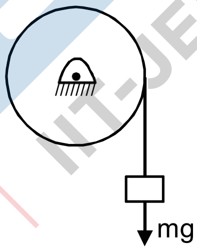
(A*) $\frac{g}{2}$ (B) $\frac{5g}{6}$ (C) g (D) $\frac{2g}{3}$

Sol. $mg - T = ma$

$TR = mR^2 \times \frac{a}{R}$

$mg = 2ma$

$a = g/2$



87. During the propagation of electromagnetic waves in a medium:

(A) Electric energy density is half of the magnetic energy density

(B*) Electric energy density is equal to the magnetic energy density

(C) Both electric and magnetic energy densities are zero

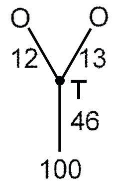
(D) Electric energy density is double of the magnetic energy density

88. Three rods of Copper, Brass and Steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100ºC where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is

(A) 2.4 cal/s (B*) 4.8 cal/s (C) 6.0 cal/s (D) 1.2 cal/s

Note: The answer is wrong because the answer is 4.8 erg/s.

Sol. $8\times\dfrac{\dfrac{(100-T)}{46}}{0.92\times 4}=\dfrac{\dfrac{T-0}{13}}{0.26\times 4}+\dfrac{\dfrac{T-0}{12}}{0.12\times 4}$



0.08 (100 – T) = 0.08T + 0.09T

8 = 0.11T

T = 40

$\dfrac{dH}{dt}$ = 60 × 0.08 = 4.8 erg/s

89. Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionised Helium ($(_2He^4)^+$) and doubly ionised lithium ($(_3Li^6)^{++}$) all have one electron around the nucleus. Consider an electron transition from n = 2 to n = 1. If the wave lengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is correct?

(A) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (B*) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

(C) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$ (D) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

Sol. $\dfrac{1}{\lambda}=RZ^2\times\dfrac{3}{4}$

$\dfrac{1}{\lambda_1}=\dfrac{3R}{4}$

$\lambda_1=\dfrac{4}{3R}=\lambda_2$

$\lambda_3=\dfrac{4}{3R\times 4}=\dfrac{\lambda_2}{4}$

$\lambda_4=\dfrac{4}{3R\times 9}=\dfrac{\lambda_1}{9}$

90. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is:

(A) $\dfrac{1}{2}\left(aL^2+bL^3\right)$ (B*) $\dfrac{aL^2}{2}+\dfrac{bL^3}{3}$ (C) $\dfrac{1}{2}\left(\dfrac{aL^2}{2}+\dfrac{bL^3}{3}\right)$ (D) $aL^2 + bL^3$

Sol. $W=\int_0^L (ax+bx^2)dx=\dfrac{aL^2}{2}+\dfrac{bL^3}{3}$